6. cvičení - výsledky

Příklad 1.

(a) Maximum hodnoty e v bodech $[1,0]$ a minimum hodnoty $\frac{1}{e}$ v bodě $[-1,0]$.

(b) Maximum hodnoty 1 v bodě $[1,0]$ a minimum hodnoty 0 v bodech $[0,y], y \in [0,1]$.

(c) Maximum hodnoty 1 v bodech $[\pm 1,0]$ a minimum hodnoty $\frac{1}{4}$ v bodech $\left[0,\pm\frac{1}{2}\right]$.

(d) Maximum hodnoty 5 v bodech $[1,1,1],[1,-1,1],[-1,1,1],[-1,-1,1]$ a minimum hodnoty -1 v bodě $[0,0,-1]$.

(e) Maximum hodnoty $\frac{17}{4}$ v bodech $\left[\frac{3}{10},\frac{4}{5}\right],\left[-\frac{3}{10},-\frac{4}{5}\right]$ a minimum hodnoty -2 v bodech $\left[\frac{2}{5},-\frac{3}{5}\right],\left[-\frac{2}{5},\frac{3}{5}\right]$.

(f) Maximum hodnoty $\frac{1}{e}$ v bodech splňujících $x^2+y^2=1$ a minimum hodnoty 0 v bodě $[0,0]$.

(g) Maximum hodnoty $\frac{1}{2e}$ v bodě $\left[\frac{1}{2},0\right]$ a minimum hodnoty 0 v bodě $[0,0]$.

(h) Supremum hodnoty $\frac{1}{2e}$ a infimum hodnoty 0. Maxima ani minima se nenabývá.

(i) Maximum hodnoty $\frac{5}{e}$ v bodech $[0,\pm 1]$ a minimum hodnoty 0 v bodě $[0,0]$.

Příklad 2.

(a) Maximum hodnoty $\frac{1}{2}$ v bodech $[\frac{\pi}{4}+k\pi,\frac{\pi}{4}-k\pi], k\in\mathbb{Z}$ a minimum hodnoty $-\frac{1}{2}$ v bodech $[\frac{3\pi}{4}+k\pi,-\frac{\pi}{4}-k\pi], k\in\mathbb{Z}$.

(b) Maximum hodnoty -1 v bodě $[0,0]$ a minimum hodnoty -19 v bodě $[0,3]$.

(c) Supremum neexistuje a minimum hodnoty -14 v bodě $[-1,-2,3]$.

(d) Maximum hodnoty $\sqrt{2}$ v bodě $\left[\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ a minimum hodnoty $-\sqrt{2}$ v bodě $\left[-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right]$.

(e) Maximum hodnoty $\frac{5}{e}$ v bodech $[0,\pm 1]$ a minimum hodnoty 0 v bodě $[0,0]$.

(f) Maximum hodnoty 63 v bodech $[1,\pm\sqrt[6]{63}]$ a minimum hodnoty $-\frac{12\cdot 64}{7\sqrt[6]{7}}$ v bodech $\left[-\frac{2}{\sqrt[6]{7}},\pm 2\frac{\sqrt[6]{6}}{\sqrt[6]{7}}\right]$.

(g) Maximum hodnoty $\frac{\sqrt{a^2+b^2}}{ab}$ v bodě $\left[\frac{b}{\sqrt{a^2+b^2}},\frac{a}{\sqrt{a^2+b^2}}\right]$ a minimum hodnoty $-\frac{\sqrt{a^2+b^2}}{ab}$ v bodě $\left[-\frac{b}{\sqrt{a^2+b^2}},-\frac{a}{\sqrt{a^2+b^2}}\right]$.

(h) Maximum hodnoty a^2 v bodech $[\pm a,0,0]$ a minimum hodnoty 0 v bodě $[0,0,0]$.

Příklad 3.

(a) Maximum je v $\left[\frac{1}{3},-\frac{2}{3},\frac{2}{3}\right]$ a minimum v $\left[-\frac{1}{3},\frac{2}{3},-\frac{2}{3}\right]$.

(b) Maximum je v $\left[\frac{2}{\sqrt{78}},-\frac{7}{\sqrt{78}},\frac{5}{\sqrt{78}}\right]$ a minimum v $\left[-\frac{2}{\sqrt{78}},\frac{7}{\sqrt{78}},-\frac{5}{\sqrt{78}}\right]$.

(c) Maximum je v $\left[\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right]$ a minimum v $\left[-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right],\left[\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\mp\frac{1}{\sqrt{3}}\right]$.

(d) Maximum v $\left[\frac{\pi}{6},\frac{\pi}{6},\frac{\pi}{6}\right]$ a minima se nenabývá.

(e) Maximum v $\left[\frac{a}{6},\frac{a}{6},\frac{a}{6}\right]$ a minima se nenabývá.

(f) Maximum je v $\left[\pm\sqrt{\frac{7}{3}\sqrt{\frac{5}{12}}}, \sqrt{\frac{5}{12}}\right]$ a minimum v $[0,0]$.

Příklad 4.

(a) Maximum je v $\left[\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right], \left[-\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \mp\frac{1}{\sqrt{3}}\right]$ a minimum v $\left[\frac{1}{\sqrt{3}}, \mp\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right], \left[-\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right]$.

(b) Maximum je v $\left[\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right], \left[-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right], \left[-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right]$ a minimum v $\left[-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right], \left[\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right], \left[\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right]$.

(c) Maximum v $[1,1]$ a minimum v $[0,0]$.

(d) Maximum v $\left[\pm\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$ a minimum v $\left[\pm\frac{\sqrt{3}}{2}, -\frac{1}{2}\right]$.

(e) Maximum je v $\left[\frac{1}{4}\sqrt{a}\sqrt[4]{15}, \frac{1}{4}\sqrt{a}\sqrt[4]{15^3}\right]$ a minimum v $\left[-\frac{1}{4}\sqrt{a}\sqrt[4]{15}, -\frac{1}{4}\sqrt{a}\sqrt[4]{15^3}\right]$.

(f) Maximum je v $\left[\frac{1}{\sqrt{102}}, -\frac{1}{\sqrt{102}}, \frac{10}{\sqrt{102}}\right]$ a minimum v $\left[-\frac{1}{\sqrt{102}}, \frac{1}{\sqrt{102}}, -\frac{10}{\sqrt{102}}\right]$.

(g) Maximum je v $\left[\frac{9}{5}, \frac{13}{5}\right]$ a minimum v $\left[\frac{1}{5}, \frac{7}{5}\right]$.

(h) Maximum je v $\left[\frac{\sqrt{5}}{2}, 2\sqrt{5}\right]$ a minimum v $\left[-\frac{\sqrt{5}}{2}, -2\sqrt{5}\right]$.

(i) Maximum v $\left[\frac{2\sqrt{2}}{\sqrt[4]{5}}, \frac{2}{\sqrt[4]{5}}\right]$ a minimum v $\left[\frac{2\sqrt{2}}{\sqrt[4]{5}}, -\frac{2}{\sqrt[4]{5}}\right]$.

(j) Maximum v $[0,1]$ a minimum v $[0,0]$.

(k) Maximum v $\left[\frac{1\pm\sqrt{5}}{4}, \frac{1}{2}, \frac{1\mp\sqrt{5}}{4}\right]$ a minimum v $\left[\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right]$.

(l) Maximum v $\left[-\frac{1}{2}, 0\right]$ a minimum v $[-2,0]$.

(m) Maximum v $\left[\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, 0\right]$ a minimum v $\left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \pm1\right], \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \pm1\right]$.

(n) Maximum v $\left[0, \pm\frac{1}{2}\right]$ a minimum v $[0,0]$.

(o) Maximum v $\left[\pm\frac{1}{2}, \pm\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$ a minimum v $\left[\pm\frac{1}{2}, \mp\frac{1}{2}, -\frac{1}{\sqrt{2}}\right]$.

(p) Maximum v $\left[\frac{\sqrt{5}-1}{2}, 0, \sqrt{\frac{\sqrt{5}-1}{2}}\right]$ a minimum v $\left[\frac{\sqrt{5}-1}{2}, 0, -\sqrt{\frac{\sqrt{5}-1}{2}}\right]$.

(q) Maximum v $\left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ a minimum v $[0,0]$.